

Vacuum magnetic linear birefringence using pulsed fields: the BMV experiment

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In this letter we present the measurement of the vacuum magnetic birefringence obtained using the first generation setup of the BMV experiment. In particular, we detail our procedure of data acquisition and our analysis which takes into account the symmetry properties of raw data with respect to the orientation of the magnetic field and the sign of the cavity birefringence. Our current value of vacuum magnetic linear birefringence k_{CM} was obtained with about 100 magnetic pulses and a maximum field of 6.5 T. We get $k_{\text{CM}} \sim (-7.4 \pm 8.7) \times 10^{-21} \text{ T}^{-2}$ at 3σ confidence level. Our result is a clear validation of our innovative experimental method.

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It is known since the beginning of the 20th century that any medium shows a linear birefringence in the presence of a transverse external magnetic field B . This effect is usually known as the Cotton-Mouton (CM) effect ([1] and references therein). The existence of such a magnetic linear birefringence has also been predicted in vacuum around 1970 in the framework of Quantum ElectroDynamics. It is one of the non linear optical effects described by the Heisenberg-Euler effective lagrangian ([2] and references therein) and it can be seen as the result of the interaction of the external magnetic field with quantum vacuum fluctuations. In a vacuum therefore the index of refraction n_{\parallel} of light polarized parallel to B is expected to be different from the index of refraction n_{\perp} of light polarized perpendicular to B such that $\Delta n_{\text{CM}} = n_{\parallel} - n_{\perp} = k_{\text{CM}} B^2$. At the first order in the fine structure constant α , k_{CM} can be written as $k_{\text{CM}} = 2\alpha^2 \hbar^3 / 15\mu_0 m_e^4 c^5$ [2], with \hbar the Planck constant over 2π , m_e the electron mass, c the speed of light in vacuum, and μ_0 the magnetic constant. Using the CODATA recommended values for fundamental constants [3], one obtains $k_{\text{CM}} \sim 4.0 \times 10^{-24} \text{ T}^{-2}$.

In spite of several experimental attempts, the proof of such a very fundamental QED prediction is still lacking [2]. All recent experiments, both completed or running, measure Δn_{CM} via the ellipticity ψ induced on a linearly polarized light propagating in the birefringent vacuum:

$$\psi = \pi k_{\text{CM}} \frac{L_B}{\lambda} B^2 \sin 2\theta, \quad (1)$$

where λ is the light wavelength, L_B is the path length in the magnetic field, and θ is the angle between the light polarization and the birefringence axis adjusted to 45° [2]. This equation clearly shows that the critical experimental parameter is the product $B^2 L_B$. In order to increase the ellipticity to be measured, one usually uses an optical cavity to store light in the magnetic field region as long as possible. The total acquired ellipticity Ψ is linked to the ellipticity ψ acquired in the absence of cavity and depends on the cavity finesse F as: $\Psi = \frac{2F}{\pi} \psi$.

After the theoretical predictions of the 70s [4, 5], a first measurement of the k_{CM} value was published by the BFRT collaboration. It was based on a superconducting magnet providing a maximum field of 3.9 T, and a multipass optical cavity. Spurious signals were always present (see Table V(b) in [6]). Final results gave $k_{\text{CM}} = (2.2 \pm 0.9) \times 10^{-19} \text{ T}^{-2}$ at 3σ confidence level for 34 reflections inside the cavity, and $k_{\text{CM}} = (3.2 \pm 1.4) \times 10^{-19} \text{ T}^{-2}$ for 578 reflections. All over our letter, we give error bars at 3 sigma since it corresponds to the confidence level usually indicating an evidence for a non zero signal. In 2008 a new measurement was published by the PVLAS collaboration using a Fabry-Perot optical cavity and a superconducting magnet providing a 2.3 T field: $k_{\text{CM}} = (1.4 \pm 2.4) \times 10^{-20} \text{ T}^{-2}$ at 3σ [7]. The same experiment at 5 T gave $k_{\text{CM}} = (2.7 \pm 1.2) \times 10^{-20} \text{ T}^{-2}$ at 3σ . More recently a new version of PVLAS apparatus based on two 2.5 T permanent magnets and a Fabry-Perot optical cavity has reached a noise floor corresponding to $k_{\text{CM}} = 1.3 \times 10^{-20} \text{ T}^{-2}$ at 3σ , but only when no spurious signal was observed [8]. This clearly shows that vacuum CM measurements are very challenging and that experimentalists have to focus not only on getting the best optical sensitivity and maximizing the signal to be measured, but also on minimizing all the unwanted systematic effects by decoupling the apparatus from their sources and by performing an appropriate data analysis.

In this letter we present a measurement of k_{CM} obtained using the first generation setup of the BMV (*Biréfringence Magnétique du Vide*) experiment at the High Magnetic Field National Laboratory of Toulouse, France (LNCMI-T) [9]. The novelty of this experiment is the use of pulsed magnetic fields. This method provides the highest magnetic fields in terrestrial laboratories without destroying the coil itself [2]. Our apparatus is also based on the use of a Fabry-Perot cavity among the sharpest in the world [10]. We present our procedure of data acquisition and our analysis that takes into account the symmetry properties of raw data with respect to the orientation of the magnetic field and the sign of

the cavity birefringence. We are then able to measure systematic effects and to overcome them. Our current value of k_{CM} was obtained with 101 magnetic pulses and a maximum field of 6.5 T. This result is compatible at 3σ with the expected value for vacuum and corresponds to one of the most precise measurement ever realized. It is therefore a clear validation of our innovative experimental method.

Our experimental setup is described in Refs. [9, 10]. As shown in Fig. 1, 30 mW of a linearly polarized Nd:YAG laser ($\lambda = 1064$ nm) is injected into the 2 meters-long high finesse Fabry-Perot cavity consisting of the mirrors M_1 and M_2 . To this end, the laser passes through an electro-optic modulator (EOM) creating sidebands at 10 MHz. We analyze the beam reflected by the cavity on the photodiode Ph_r . This signal is used to lock the laser frequency to the cavity resonance frequency using the Pound-Drever-Hall method [11].

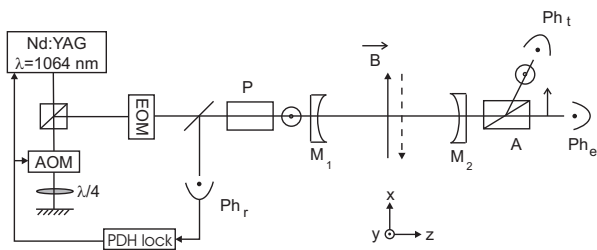


FIG. 1: Experimental setup. A Nd-YAG laser is frequency locked to the Fabry-Perot cavity made of mirrors M_1 and M_2 . The laser beam is linearly polarized by the polarizer P and analyzed with the polarizer A . This analyzer allows to extract the extraordinary beam sent on photodiode Ph_e as well as the ordinary beam sent on photodiode Ph_t . The beam reflected by the cavity analyzed on the photodiode Ph_r is used for the cavity locking. A transverse magnetic field \vec{B} can be applied inside the cavity in order to study the magnetic birefringence of the medium. \vec{B} can be set parallel (solid arrow) or antiparallel (dashed arrow) to the x -direction. EOM = electro-optic modulator; AOM = acousto-optic modulator, PDH = Pound-Drever-Hall.

Pulsed magnets can in principle provide fields of several tens of Teslas. The principle of these magnets is described in Refs. [9, 13]. Our apparatus consists of one magnet with an equivalent length of $L_B = 0.137$ m inserted inside the cavity. Data have been taken with a maximum magnetic field of 6.5 T reached within 2 ms while the total duration of a pulse is less than 10 ms as shown in Fig. 2. Moreover, the high voltage connections can be remotely switched to reverse \vec{B} in order to set it parallel or antiparallel to the x -direction. The maximum repetition rate is 6 pulses per hour.

We infer the cavity finesse from the measurement of the photon lifetime [10]. Its value is regularly checked during data taking and we get $F = 445\,000$ with a relative variation that does not exceed 6% at the 3σ level.

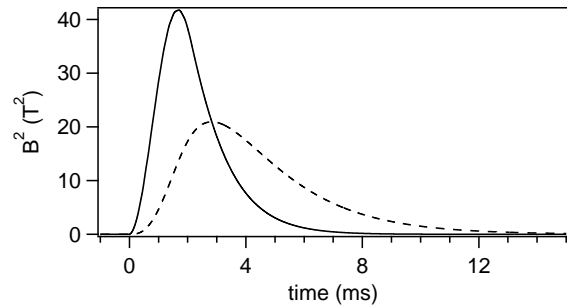


FIG. 2: Square of the magnetic field amplitude as a function of time. Solid black curve: B^2 , dashed curve: B_f^2 .

Before entering the Fabry-Perot cavity light is precisely polarized by the polarizer P . The beam transmitted by the cavity is then analyzed by the analyzer A crossed at maximum extinction. Both polarizations are extracted: parallel and perpendicular to P . The extraordinary ray (power I_e) is detected by the photodiode Ph_e , while the ordinary ray is detected by Ph_t (power I_t). Then, one gets:

$$\frac{I_e(t)}{I_{t,f}(t)} = \sigma^2 + [\Gamma + \Psi(t)]^2, \quad (2)$$

$$\simeq \sigma^2 + \Gamma^2 + 2\Gamma\Psi(t) \text{ for } \Psi \ll \Gamma. \quad (3)$$

The polarizer extinction ratio σ^2 is about 7×10^{-7} . The ellipticity induced by the external magnetic field is proportional to B_f^2 . We calculate the quantities $I_{t,f}$ and B_f^2 from I_t and B^2 taking into account the first-order low-pass filtering of the cavity as explained in Ref. [12]. Time profile of B_f^2 is plotted in dashed curve in Fig. 2 showing the attenuation and the shift of the maximum due to this cavity filtering.

The total static ellipticity Γ is due to the mirror intrinsic phase retardation [14]. Mirrors can be regarded as wave plates and thus one can set the value of Γ by adjusting their respective orientation [15, 16]. To reach the best sensitivity, one needs Γ to be as small as possible [9]. For our experiment, we adjust both mirrors' orientation in order to have $\Gamma^2 \simeq \sigma^2$. This adjustment also allows to choose the sign of Γ which can be changed between pulses.

We have calibrated our experiment using nitrogen gas [10]. Gas measurements are also used to extract the sign of Γ . For measurements in vacuum, all the optical devices from the polarizer to the analyzer are placed in an ultrahigh-vacuum chamber. During operation, the pressure inside the UHV vessel was about 10^{-7} mbar. The vacuum quality has been monitored with a residual gas analyzer.

For the measurements reported in this letter, more than a hundred pulses have been applied under vacuum.

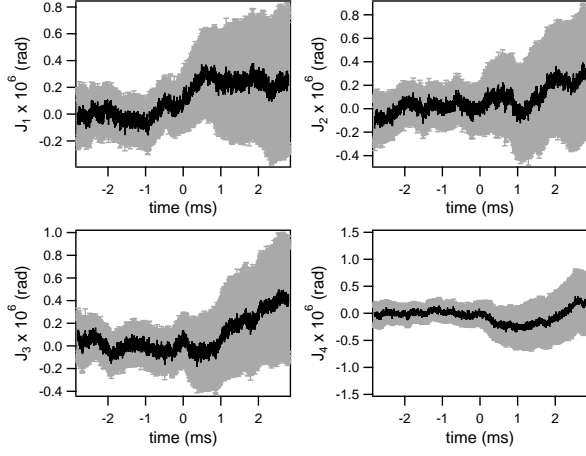


FIG. 3: Terms J calculated with more than a hundred pulses. Black: mean value; Gray: 3σ statistical uncertainties.

For each pulse, we calculate the following signal:

$$Y(t) = \frac{\frac{I_e(t)}{I_{t,t}(t)} - \sigma^2 - \Gamma^2}{2|\Gamma|}. \quad (4)$$

Before each pulse, we first adjust Γ to zero in order to measure σ^2 precisely. The cavity ellipticity Γ is then adjusted and finally measured a few milliseconds before the magnetic pulse. We acquire the signals with both signs of Γ and for both directions of \vec{B} (parallel to x is denoted as > 0 and antiparallel is denoted as < 0). This gives four data series: $(\Gamma > 0, B > 0)$, $(\Gamma > 0, B < 0)$, $(\Gamma < 0, B < 0)$ and $(\Gamma < 0, B > 0)$. For each series, signals calculated with Eq. (4) are averaged and denoted as $Y_{>>}$, $Y_{><}$, $Y_{<<}$ and $Y_{<>}$. The first subscript corresponds to $\Gamma > 0$ or < 0 while the second one corresponds to \vec{B} parallel or antiparallel to x .

Following Eq. (3), for an ideal experiment we would obtain:

$$Y(t) = \gamma \Psi(t). \quad (5)$$

The $\gamma = \frac{\Gamma}{|\Gamma|}$ parameter indicates the sign of Γ . Actually, one has to consider systematic effects that mimic the CM effect we want to measure. We thus derive a more general expression taking into account the symmetry properties of Y towards experimental parameters:

$$\begin{aligned} Y_{>>} &= a_{>>} S_{++} + b_{>>} S_{+-} + c_{>>} S_{--} + d_{>>} S_{-+}, \\ Y_{><} &= a_{><} S_{++} - b_{><} S_{+-} - c_{><} S_{--} + d_{><} S_{-+}, \\ Y_{<<} &= a_{<<} S_{++} - b_{<<} S_{+-} + c_{<<} S_{--} - d_{<<} S_{-+}, \\ Y_{<>} &= a_{<>} S_{++} + b_{<>} S_{+-} - c_{<>} S_{--} - d_{<>} S_{-+}. \end{aligned}$$

The S functions correspond to a given symmetry towards the sign of Γ and the direction of \vec{B} . The first subscript corresponds to Γ . First subscript $+$ (resp. $-$) indicates

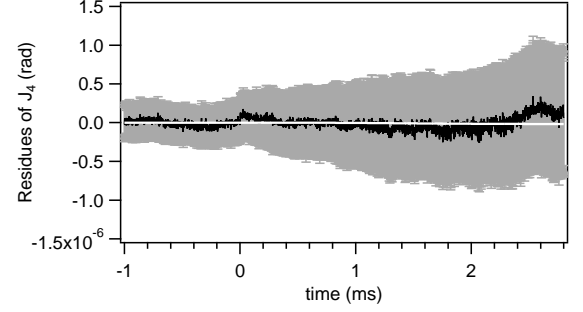


FIG. 4: Residues of J_4 . Black: mean value, gray: 3σ statistical uncertainties. J_4 is fitted by an expression proportional to a CM effect (white line).

an even (resp. odd) parity with respect to the sign of Γ . The same convention is used for the second subscript corresponding to \vec{B} . Each S function has a different physical origin. For example, effects originating outside the Fabry-Perot cavity does not change sign with Γ and therefore contribute to S_{++} and S_{+-} . Faraday effects, which are linear in the magnetic field, contribute to S_{+-} and S_{--} .

CM effect signal contributes to S_{-+} since it depends on the cavity birefringence and on the square of the B field amplitude as shown in Eqns. (1) and (5). They can have several origins: vacuum, residual gases or cavity mirrors. Concerning residual gases, most important contributions come from N_2 and O_2 leading to a k_{CM} of $1.5 \times 10^{-23} \text{ T}^{-2}$. An estimation of the CM effect of high-finesse dielectric mirrors has been reported in Ref. [17] with an induced ellipticity of $8 \times 10^{-10} \text{ rad.T}^{-2}$ per reflection. The transverse magnetic field at mirror position is smaller than $150 \mu\text{T}$, corresponding to $k_{CM} = 1 \times 10^{-24} \text{ T}^{-2}$. All these CM effects are of the order of the one predicted by QED for vacuum.

Parameters a , b , and c depend on the experimental adjustment from pulse to pulse and from day to day, but we can assume that their variations are small compared to their mean value: $\Delta a, \Delta b, \Delta c < \bar{a}, \bar{b}, \bar{c}$. On the other hand, the CM term d is expected to be much smaller than spurious effects: $\bar{d} \ll \Delta a, \Delta b, \Delta c$. S functions can be extracted with a combination of Y as following:

$$\begin{aligned} J_1 &= \frac{Y_{>>} + Y_{><} + Y_{<<} + Y_{<>}}{4} \simeq \bar{a} S_{++}, \\ J_2 &= \frac{Y_{>>} - Y_{><} - Y_{<<} + Y_{<>}}{4} \simeq \bar{b} S_{+-}, \\ J_3 &= \frac{Y_{>>} - Y_{><} + Y_{<<} - Y_{<>}}{4} \simeq \bar{c} S_{--}, \\ J_4 &= \frac{Y_{>>} + Y_{><} - Y_{<<} - Y_{<>}}{4} \\ &\simeq \Delta a S_{++} + \Delta b S_{+-} + \Delta c S_{--}. \end{aligned} \quad (6)$$

Experimentally, we first calculate terms $Y_{>>}$, $Y_{><}$, $Y_{<<}$ and $Y_{<>}$ as a function of time by averaging signals

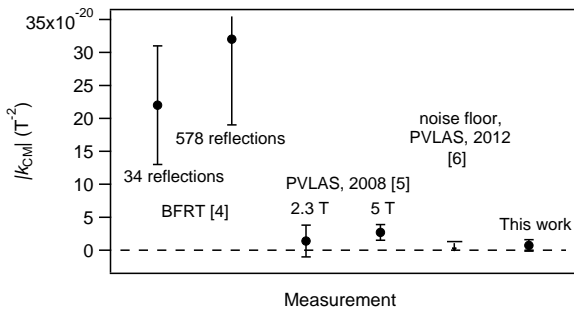


FIG. 5: Comparison of absolute reported values of the vacuum CM effect.

$Y(t)$ acquired with the same experimental parameters. We also calculate the standard deviation at any time in order to associate a statistical uncertainty to each data point. We then extract the J terms. These signals are plotted in black in Fig. 3 together with their statistical uncertainties in gray. Acoustic perturbations induce signal oscillations at about 4 ms [10]. In order to minimize the influence of this noise on our final result, we limit the integration time to 2.8 ms which corresponds to the maximum of B_f^2 .

Following Eq. (6), we fit J_4 with a linear combination of J_1 , J_2 and J_3 i.e. a linear combination of S_{++} , S_{+-} and S_{--} . Fit residues are plotted in Fig. 4. It corresponds to $\bar{d}S_{-+}$ and thus includes the CM effect we want to measure. J_4 residues is fitted by a CM effect $\alpha_{\text{CM}} B_f^2$, giving $\alpha_{\text{CM}} = (-8.5 \pm 10.0) \times 10^{-10} \text{ rad.T}^{-2}$ at 3σ and $k_{\text{CM}} = \alpha_{\text{CM}} \lambda / 2FL_B = (-7.4 \pm 8.7) \times 10^{-21} \text{ T}^{-2}$ at 3σ confidence level. The uncertainty given for k_{CM} takes into account the one due to α_{CM} and the one due to the other experimental parameters B^2 , λ , F and L_B [10]. For the sake of argument, if we neglect spurious signals and just fit, for example, the $Y_{>>}$ raw signal by a CM effect $\alpha_{>>} B_f^2$, we obtain $\alpha_{>>} = (2.35 \pm 0.12) \times 10^{-8} \text{ rad.T}^{-2}$ at 3σ . This value is absolutely incompatible with the expected one, clearly showing that spurious signal subtraction is essential to get accurate values.

Our k_{CM} value is compatible with the expected one for vacuum and it is one of the most precise value ever realized as shown in Fig. 5. It definitely validates our experimental method based on pulsed fields proving that the sensitivity obtained in a single pulse compensates the loss of duty cycle. We reach a noise floor that is similar to the one of PVLAS collaboration in 2012 obtained with an integration time of 8192 s [8].

To reach the QED value, the needed improvement has to be of three orders of magnitude. This is not conceivable with our first-generation experiment. Our strategy is therefore to increase the magnetic field thanks to the pulsed technology. At the moment, we have $B^2 L_B = 5.8 \text{ T}^2 \text{m}$ but we have built a pulsed coil pro-

tototype that has already reached a $B^2 L_B$ higher than $300 \text{ T}^2 \text{m}$. Two coils of this type will be inserted in the experiment in the near future. This essential milestone really makes the vacuum birefringence measurement within our reach. On the other hand, our analysis has allowed us to identify some systematic effects. Obviously, a special care will be devoted to limit them in order to improve the accuracy. Furthermore this accuracy could be limited by spurious effects having the symmetry properties necessary to contribute to J_4 function. For example, function S_{-+} also includes terms such as $B \frac{dB}{dt}$ that can be due to the variation of the magnetic flux giving rise to a current flow in conductive materials, such as mirror mounts or vacuum tubes, that couples with B and leads to a force. This force can act on the cavity mirrors resulting in a change of Γ , thus proportional to its sign. To separate this kind of effect from the CM one, one should in principle change the pulse duration while keeping the same maximum field. This could be implemented in our current magnetic field installation.

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